Optimization of White Layer Cake Formulations by a Multiple-Factor Experimental Design[1]

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ABSTRACT

The effects of simultaneous variation of seven full-formula white layer cake ingredients were studied in a central-composite design of the Box-Wilson type. Six linearly independent ingredient ratios were defined, with five levels of each ratio establishing the experimental space. Seventy-seven treatment combinations were baked to sample the responses: cake volume, top-contour shape, and internal score. Multiple-regression analysis provided second-order equations for computing response surfaces. In the analysis of variance, significant effects on cake volume were found for water, sugar, leavening, and flour ratios and for five interaction terms. Top contour was responsive to the water × sugar interaction; internal appearance was affected by leavening, egg albumen, and flour ratios, and by several interactions. Some response surfaces are presented to show the range of the variables in which superior cake performance is indicated. Test baking confirmed the performance predicted by the equations within the experimental range. Stationary points for the system were located by mathematical methods and tested.

An earlier paper (1) described the application of a central-composite experimental design of the Box-Wilson type (2,3) to study baking responses of a simplified layer-cake formula. In that work the five basic ingredients and the central composition were identical with the lean (research) formulation of Kissell (4). Although results in terms of individual ingredients were complicated by the necessary definition of ingredient ratios functioning as independent variables, much information was obtained relating the concentration of batter components to cake volume and contour. Performance of the design's center composition was modified greatly by changing either the leavening or the sugar ratios, or both, and was affected by variations in flour and shortening ratios. The areas of formulation were indicated in which greater layer volumes and satisfactory contours were expected, and where undesirable "dipping" or "peaking" occurs.

A second multiple-factor experiment was designed to assess the contributions of egg albumen and nonfat dry milk as structural variables, and to meet the objection that the research cake method (4) does not yield a commercial-type cake. Salt and vanilla extract were included in constant amounts and a new, untested center-point composition was derived from a commercial baker's recipe. The experiment was descriptive of a seven-ingredient system over the selected range of variability, emphasizing compositions that optimized quality factors.

Smith and Rose (5) applied multiple-factor analysis to study the effects of water, flour, and shortening variables on pie-crust consistency and specific volume. Wilson and Donelson (6), in another response-surface study, located

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areas of interaction between chlorine-treatment level and the quantity of liquid required to obtain optimum cake with the research formula (4). Both studies employed surfaces generated by least-squares fit of limited data to predict variable combinations with superior performance.

MATERIALS AND METHODS

Test-Baking Method

Ingredients and procedures were similar to those used in the basic-formulation study (1). Dry ingredients, used to simulate package-mix technique, included extra-fine (Bakers' Special) sucrose, spray-dried egg albumen, and nonfat dry milk. The same high-ratio shortening and commercial premium cake patent were used as in the earlier work. The flour analysis (14% moisture basis) was: ash, 0.31%; protein, 7.7%; and pH, 4.5.

Table I lists ingredients in descending order of assumed importance as quality variables. Salt and vanilla are omitted from the percentage composition of batter because they were not involved in ratio computation.

TABLE I

FORMULA COMPOSITION AT THE DESIGN CENTER POINT

Ingredient	Weight	Flour Basis	Batter Basis	Ingredient	Weight	Flour Basis	Batter Basis
	g.	%	%		g.	%	%
A. Water	171.5	114.3	29.3	E. Flour	150.0	100.0	25.7
B. Sugar	180.0	120.0	30.8	F. Nonfat dry milk	15.0	10.0	2.6
C. Baking powder	9.4	6.3	1.6	G. Shortening	50.0	33.3	8.6
D. Dry egg albumen	8.5	5.7	1.4	Salt	4.0	2.7	......
				Vanilla	3.0	2.0	......

The two-stage mixing procedure described in previous studies (1,4) was adapted as follows: Dry ingredients were blended and added to a 3-qt. bowl, along with shortening and 100 ml. of distilled water. Batters were mixed on a Hobart Model C-100 with paddle for 0.5 min. at low speed and 3.0 min. at medium. The remaining water to fulfill the treatment plus 3.0 ml. of vanilla were added in the second stage. Batters were blended for 0.5 min. at low speed, finished for 1.5 min. at medium, and divided by scaling 240 g. into 6 × 1½-in. bottom-lined steel pans. Layers were baked at 375°F. (190°C.) for 20 min. in a conditioned, reel-type, electric oven with control to ±1.0°F.

After cooling, layers were measured by seed displacement and contours coded on a numerical basis ranging from 1.0 for greatly sunken to 10.0 for highly peaked crowns. Internal scores were judged on a 0–4 scale for the factors: cell size, cell-wall thickness, and uniformity of cell distribution. The sum was a relative total score for visual properties.

Experimental Design

The design of the previous experiment (1) was expanded to include egg albumen and milk solids as variables and salt and vanilla as constants, and used a different starting composition. Six x_i ingredient ratios were chosen, observing the constraint that all variables be linearly independent, as discussed by Hackler *et al.* (7) and others (1,8). Ratios were selected in descending order of anticipated effect on cake quality, to avoid confounding two ingredients with large responses in the final (x_6) ratio. The variables were evaluated for the center-point composition (Table I) as follows: Let

x_1 = wt. of water/wt. of remainder = A/(B + C + D + E + F + G) = 0.415
x_2 = wt. of sugar/remainder = B/(C + D + E + F + G) = 0.773
x_3 = wt. of leavening/remainder = C/(D + E + F + G) = 0.042
x_4 = wt. of egg albumen/remainder = D/(E + F + G) = 0.040
x_5 = wt. of flour/remainder = E/(F + G) = 2.308
x_6 = wt. of milk solids/wt. of shortening = F/G = 0.300

In Table II are given the increments of variation for each variable, spaced around the center-point ratios, along with equations relating actual

TABLE II
ACTUAL VALUE OF CODED LEVELS OF x_i RATIOS

x_i	± INCREMENT	CODED LEVEL X_i				
		1	2	3 (Normal)	4	5
x_1	0.029	0.357	0.386	0.415	0.444	0.473
x_2	0.064	0.645	0.709	0.773	0.837	0.901
x_3	0.010	0.022	0.032	0.042	0.052	0.062
x_4	0.010	0.020	0.030	0.040	0.050	0.060
x_5	0.230	1.848	2.078	2.308	2.538	2.768
x_6	0.080	0.140	0.220	0.300	0.380	0.460

where: $X_1 = (x_1 - 0.328)/0.029$ $X_3 = (x_3 - 0.012)/0.010$ $X_5 = (x_5 - 1.618)/0.230$
$X_2 = (x_2 - 0.581)/0.064$ $X_4 = (x_4 - 0.010)/0.010$ $X_6 = (x_6 - 0.060)/0.080$

and coded ratios. By substitution in these equations, batter compositions were coded for solutions of the multiple-regression (prediction) equations, or points of interest were decoded for test-baking.

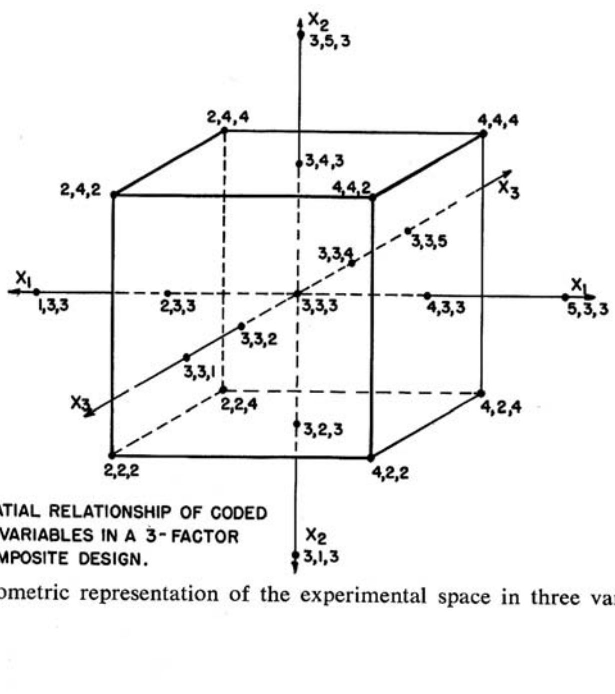


Fig. 1. Isometric representation of the experimental space in three variables.

TABLE III

Coded Levels of Ingredient Ratios for Experimental Treatments, Ingredient Proportions, and Resultant Cake Quality Data

Treatment	Ratio Level						Ingredient							Baking Data		
	X_1	X_2	X_3	X_4	X_5	X_6	A Water	B Sugar	C B.P.	D Egg	E Flour	F DMS	G Short.	Cake Vol.	Contour Score[a]	Internal Score
							%	%	%	%	%	%	%	cc.		
1	2	2	2	2	2	2	27.85	29.93	1.31	1.19	26.82	2.32	10.58	609	7.3	9.0
2	4	2	2	2	2	2	30.75	28.73	1.26	1.14	25.73	2.24	10.15	551	7.5	9.0
3	2	4	2	2	2	2	27.85	32.87	1.22	1.11	24.94	2.16	9.85	548	4.0	7.5
4	4	4	2	2	2	2	30.75	31.55	1.17	1.06	23.95	2.08	9.44	557	9.0	10.3
5	2	2	4	2	2	2	27.85	29.93	2.09	1.17	26.30	2.29	10.37	571	2.5	4.8
6	4	2	4	2	2	2	30.75	28.73	2.00	1.12	25.25	2.19	9.96	600	6.0	7.0
7	2	4	4	2	2	2	27.85	32.87	1.94	1.09	24.47	2.13	9.65	526	1.5	3.0
8	4	4	4	2	2	2	30.75	31.55	1.86	1.04	23.49	2.04	9.27	535	3.0	6.0
9	2	2	2	4	2	2	27.85	29.93	1.31	1.95	26.30	2.29	10.37	600	7.0	8.5
10	4	2	2	4	2	2	30.75	28.73	1.26	1.87	25.24	2.19	9.96	579	8.0	9.0
11	2	4	2	4	2	2	27.85	32.87	1.22	1.81	24.47	2.13	9.65	565	4.0	7.5
12	4	4	2	4	2	2	30.75	31.55	1.17	1.74	23.49	2.04	9.26	554	8.0	9.0
13	2	2	4	4	2	2	27.85	29.93	2.09	1.91	25.80	2.24	10.18	565	3.0	5.0
14	4	2	4	4	2	2	30.75	28.73	2.00	1.83	24.77	2.15	9.77	586	4.5	6.8
15	2	4	4	4	2	2	27.85	32.87	1.94	1.78	24.01	2.09	9.46	513	1.5	3.5
16	4	4	4	4	2	2	30.75	31.55	1.86	1.71	23.04	2.00	9.09	547	3.0	5.8
17	2	2	2	2	4	2	27.85	29.93	1.31	1.19	28.49	2.02	9.21	618	7.5	8.8
18	4	2	2	2	4	2	30.75	28.73	1.26	1.14	27.35	1.94	8.83	585	7.8	8.8
19	2	4	2	2	4	2	27.85	32.87	1.22	1.11	26.51	1.88	8.56	601	6.0	9.3
20	4	4	2	2	4	2	30.75	31.55	1.17	1.06	25.44	1.81	8.22	581	9.0	9.5
21	2	2	4	2	4	2	27.85	29.93	2.09	1.17	27.95	1.99	9.02	573	3.0	5.3
22	4	2	4	2	4	2	30.75	28.73	2.00	1.12	26.83	1.90	8.67	616	6.0	6.5
23	2	4	4	2	4	2	27.85	32.87	1.94	1.09	26.00	1.85	8.40	521	2.0	4.3
24	4	4	4	2	4	2	30.75	31.55	1.86	1.04	24.96	1.78	8.06	567	3.5	5.5
25	2	2	2	4	4	2	27.85	29.93	1.31	1.95	27.94	1.98	9.04	619	7.0	8.0

(Continued)

TABLE III (Continued)

Treatment	Ratio Level X1	X2	X3	X4	X5	X6	Ingredient A Water	B Sugar	C B.P.	D Egg	E Flour	F DMS	G Short.	Baking Data Cake Vol.	Contour Score[a]	Internal Score
							%	%	%	%	%	%	%	cc.		
26	4	2	2	4	4	2	30.75	28.73	1.26	1.87	26.82	1.90	8.67	577	7.0	9.3
27	2	4	2	4	4	2	27.85	32.87	1.22	1.81	26.00	1.85	8.40	585	6.0	8.8
28	4	4	2	4	4	2	30.75	31.55	1.17	1.74	24.96	1.77	8.06	573	8.5	10.3
29	2	2	4	4	4	2	27.85	29.93	2.09	1.91	27.39	1.95	8.86	578	3.0	5.0
30	4	2	4	4	4	2	30.75	28.73	2.00	1.83	26.32	1.87	8.50	616	6.0	6.3
31	2	4	4	4	4	2	27.85	32.87	1.94	1.78	25.51	1.81	8.23	520	2.0	4.0
32	4	4	4	4	4	2	30.75	31.55	1.86	1.71	24.48	1.74	7.91	560	3.5	5.5
33	2	2	2	2	2	4	27.85	29.93	1.31	1.19	26.82	3.55	9.34	603	6.8	8.8
34	4	2	2	2	2	4	30.75	28.73	1.26	1.14	25.73	3.41	8.98	581	8.0	9.0
35	2	4	2	2	2	4	27.85	32.87	1.22	1.11	24.94	3.30	8.71	577	4.5	7.8
36	4	4	2	2	2	4	30.75	31.55	1.17	1.06	23.95	3.17	8.35	566	8.5	9.5
37	2	2	4	2	2	4	27.85	29.93	2.09	1.17	26.30	3.49	9.18	562	2.5	4.0
38	4	2	4	2	2	4	30.75	28.73	2.00	1.12	25.25	3.35	8.81	600	4.5	6.0
39	2	4	4	2	2	4	27.85	32.87	1.94	1.09	24.47	3.25	8.55	516	1.5	3.3
40	4	4	4	2	2	4	30.75	31.55	1.86	1.04	23.49	3.12	8.21	543	3.0	4.8
41	2	2	2	4	2	4	27.85	29.93	1.31	1.95	26.30	3.49	9.18	609	7.0	9.0
42	4	2	2	4	2	4	30.75	28.73	1.26	1.87	25.24	3.35	8.81	593	7.0	9.3
43	2	4	2	4	2	4	27.85	32.87	1.22	1.81	25.47	3.25	8.55	570	4.5	7.5
44	4	4	2	4	2	4	30.75	31.55	1.17	1.74	23.49	3.11	8.18	566	8.0	9.5
45	2	2	4	4	2	4	27.85	29.93	2.09	1.91	25.80	3.42	9.00	562	3.0	4.8
46	4	2	4	4	2	4	30.75	28.73	2.00	1.83	24.77	3.28	8.63	590	4.5	6.0
47	2	4	4	4	2	4	27.85	32.87	1.94	1.78	24.01	3.18	8.37	505	1.5	3.0
48	4	4	4	4	2	4	30.75	31.55	1.86	1.71	23.04	3.05	8.04	535	3.0	4.8
49	2	2	2	2	4	4	27.85	29.93	1.31	1.19	28.49	3.09	8.13	617	7.5	7.8
50	4	2	2	2	4	4	30.75	28.73	1.26	1.14	27.35	2.96	7.79	578	7.5	7.5
51	2	4	2	2	4	4	27.85	32.87	1.22	1.11	26.51	2.88	7.58	606	6.8	7.5
52	4	4	2	2	4	4	30.75	31.55	1.17	1.06	25.44	2.77	7.29	587	8.5	9.5

(Continued)

TABLE III (Continued)

Treat-ment	Ratio Level						Ingredient							Baking Data		
	X_1	X_2	X_3	X_4	X_5	X_6	A Water	B Sugar	C B.P.	D Egg	E Flour	F DMS	G Short.	Cake Vol.	Contour Score[a]	Internal Score
							%	%	%	%	%	%	%	cc.		
53	2	2	4	2	4	4	27.85	29.93	2.09	1.17	27.95	3.03	7.97	589	3.0	4.3
54	4	2	4	2	4	4	30.75	28.73	2.00	1.12	26.83	2.91	7.66	612	5.8	5.8
55	2	4	4	2	4	4	27.85	32.87	1.94	1.09	26.00	2.83	7.45	519	2.0	3.0
56	4	4	4	2	4	4	30.75	31.55	1.86	1.04	24.96	2.71	7.13	560	3.5	5.3
57	2	2	2	4	4	4	27.85	29.93	1.31	1.95	27.94	3.03	7.97	621	7.3	8.8
58	4	2	2	4	4	4	30.75	28.73	1.26	1.87	26.82	2.91	7.66	584	7.5	8.3
59	2	4	2	4	4	4	27.85	32.87	1.22	1.81	26.00	2.83	7.45	596	5.5	8.3
60	4	4	2	4	4	4	30.75	31.55	1.17	1.74	24.96	2.71	7.13	574	8.3	9.0
61	2	2	4	4	4	4	27.85	29.93	2.09	1.91	27.39	2.99	7.87	577	3.0	4.5
62	4	2	4	4	4	4	30.75	28.73	2.00	1.83	26.32	2.86	7.52	601	4.0	6.0
63	2	4	4	4	4	4	27.85	32.87	1.94	1.78	25.51	2.77	7.29	527	2.0	3.3
64	4	4	4	4	4	4	30.75	31.55	1.86	1.71	24.48	2.65	7.00	548	3.0	5.3
65	1	3	3	3	3	3	26.31	32.13	1.68	1.53	26.75	2.68	8.92	557	2.0	5.3
66	5	3	3	3	3	3	32.11	29.60	1.54	1.41	24.66	2.47	8.21	584	7.5	9.5
67	3	1	3	3	3	3	29.33	27.71	1.73	1.59	27.66	2.77	9.21	634	7.3	8.8
68	3	5	3	3	3	3	29.33	33.49	1.50	1.37	23.94	2.39	7.98	550	3.0	5.8
69	3	3	1	3	3	3	29.33	30.81	0.86	1.50	26.16	2.62	8.72	534	7.5	9.8
70	3	3	5	3	3	3	29.33	30.81	2.33	1.44	25.18	2.52	8.39	518	1.5	3.0
71	3	3	3	1	3	3	29.33	30.81	1.61	0.75	26.16	2.62	8.72	622	6.0	7.3
72	3	3	3	5	3	3	29.33	30.81	1.61	2.17	25.17	2.52	8.39	602	4.8	6.8
73	3	3	3	3	1	3	29.33	30.81	1.61	1.47	23.87	2.98	9.93	576	3.5	7.0
74	3	3	3	3	5	3	29.33	30.81	1.61	1.47	27.02	2.25	7.51	618	6.5	6.8
75	3	3	3	3	3	1	29.33	30.81	1.61	1.47	25.66	1.37	9.75	594	6.5	8.0
76	3	3	3	3	3	5	29.33	30.81	1.61	1.47	25.66	3.50	7.62	608	5.0	6.5
77	3	3	3	3	3	3	29.33	30.81	1.61	1.47	25.66	2.57	8.55	605	6.0	7.8

[a] Contour description: 10.0 = highly peaked; 9.0 = peaked; 8.0 = rounded-peaked; 7.0 = rounded-normal; 6.0 = rounded-flat; 5.0 = very slightly sunken; 4.0 = slightly sunken; 3.0 = sunken; 2.0 = greatly sunken; 1.0 = extremely sunken.

The design was derived from a 2^6 factorial arrangement described by Cochran and Cox (9). All combinations of the variables at the 2- and 4-levels (64 treatments) were included, along with each ratio in turn at the 1- and 5-level, all other variables being held at the 3-level (12 treatments). One batter with all ratios at the 3-level was baked daily, giving 14 observations of the center point. All treatments were randomized and baked in duplicate. The experiment was reviewed and resultant data were processed by Biometrical Services, Agricultural Research Service, USDA, Beltsville, Maryland.[2]

Figure 1 is an orientation of the experimental space in terms of the first three variables at five levels.

Ratios X_1, X_2, and X_3 are increasing from level 1, through the face of the cube (level 2) to the design center (level 3), and on to the limit of variation (level 5). The combinations of 2- and 4-levels are represented as the corners of the cube and account, in all, for 64 experimental points.

Computing the Treatments

As in the basic ingredient study (1), percentage quantities of each batter component were obtained by systematic solution of the following equations relating ingredients and actual x_i ratios from Table II.

$$A = 100x_1/(1 + x_1)$$
$$B = x_2\,(100 - A)/(1 + x_2)$$
$$C = x_3\,(100 - A - B)/(1 + x_3)$$
$$D = x_4\,(100 - A - B - C)/(1 + x_4)$$
$$E = x_5\,(100 - A - B - C - D)/(1 + x_5)$$
$$F = x_6\,(100 - A - B - C - D - E)/(1 + x_6)$$
$$G = F/x_6$$

Finally, ingredient weights were obtained as the product of these percentages and the constant batter weight of 584.4 g. All flour weights were adjusted to 14% moisture and the differential was added to the calculated amount of water. Working liquid levels were reduced by 3.0 ml. to compensate for the liquid vanilla.

Table III contains ingredient levels for all treatments, along with resultant mean data for cake volume, contour description, and internal score. Individual data were analyzed for sources of variance and fitted by the method of least squares to polynomial equations.

RESULTS AND DISCUSSION

Visual evidence of quality responses for cakes with low and high levels of each single-ingredient ratio (treatments 65 through 76) are compared in Fig. 2 with the center point (treatment 77). The range of variation for the water ratio, X_1, sugar, X_2, leavening, X_3, and flour, X_5, produced marked effects on cake properties. The responses of egg albumen, X_4, and nonfat dry milk, X_6, ratios were less pronounced. Six representative sections of cake with normal (level 3) composition show the tendency toward sunken or flat

[2] Koch, E. James; private communications.

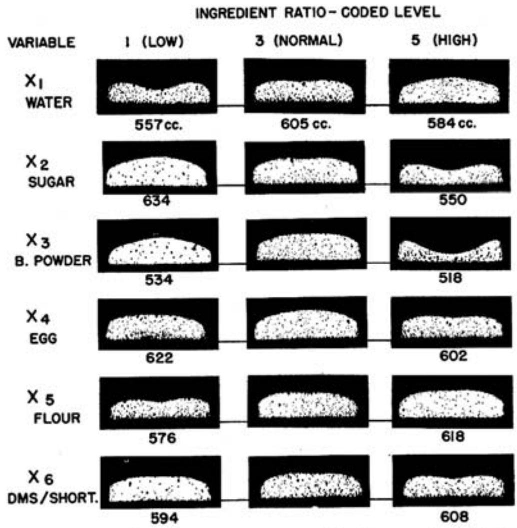


Fig. 2. Layer contour and volume responses to the main effects of each ingredient ratio at low, normal, and high coded levels. All X_i variables except the one named are held at the center (3) level. Volume of the center (normal) composition is the mean of 14 determinations.

contours obtained with the unbalanced center composition. The mean volume of 605 cc. resulted from 14 replications of the treatment with volumes ranging from 595 to 620 cc., depending on contour. Mediocre performance of the design center emphasized the ability of the method to indicate directions leading to optimum conditions.

Significant terms from the analyses of variance, summarized in Table IV, include main effects on cake volume for leavening, X_3, and flour, X_5, ratios; quadratic terms for water, X_1, sugar, X_2, and leavening, X_3; and 5-interaction terms involving those ratios. The water × sugar interaction, X_1X_2, appeared

TABLE IV

SUMMARY OF SIGNIFICANT TERMS FROM ANALYSIS OF VARIANCE

Independent Variable X_i	t-Test for Significance: Volume	Contour	Internal Score	Independent Variable X_i	t-Test for Significance: Volume	Contour	Internal Score
Linear				Interaction			
X_3	12.19**	n.s.	n.s.	X_1X_2	3.47**	2.17*	5.21**
X_5	2.82**	n.s.	2.32*	X_1X_3	16.23**	n.s.	5.40**
X_6	2.39*	n.s.	n.s.	X_1X_5	n.s.	n.s.	–2.75**
Quadratic				X_2X_3	–10.29**	n.s.	–7.48**
				X_2X_5	2.04*	n.s.	4.07**
X_1^2	– 7.64**	n.s.	n.s.	X_3X_6	–3.42**	n.s.	n.s.
X_2^2	– 2.95**	n.s.	n.s.	X_4X_6	n.s.	n.s.	2.18*
X_3^2	–17.46**	n.s.	–5.49**	X_5X_6	n.s.	n.s.	–2.37**
X_4^2	n.s.	n.s.	–2.76**				
X_5^2	n.s.	n.s.	–3.31**				

Volume	Contour	Internal Score
R^2 = 0.9450	R^2 = 0.9222	R^2 = 0.9528
R = 0.9720**	R = 0.9603**	R = 0.9761**

to be the only significant term for cake contour. Internal scores were most influenced by the quadratic terms of leavening, X_3, egg, X_4, and flour, X_5, ratios and by 7-interaction terms. The table also includes R^2, the measure of fit of data to the response surface, and the multiple regression coefficient, R, which were highly significant for all responses.

Multiple-regression equations were generated relating cake volume, contour, and internal score to coded levels of the variables, with decimals rounded to two places:

Cake volume:

$$Y = 372.29 + 7.64X_1 + 15.94X_2 + 106.82X_3 - 0.90X_4 + 24.72X_5 + 20.92X_6 - 8.66X_1^2 - 3.35X_2^2 - 19.78X_3^2 + 1.72X_4^2 - 1.97X_5^2 - 0.97X_6^2 + 2.84X_1X_2 + 13.26X_1X_3 + 0.16X_1X_4 - 0.46X_1X_5 - 0.15X_1X_6 - 8.40X_2X_3 - 0.99X_2X_4 + 1.66X_2X_5 + 0.04X_2X_6 - 1.26X_3X_4 - 0.66X_3X_5 - 2.79X_3X_6 - 1.04X_4X_5 - 0.57X_4X_6 - 1.07X_5X_6$$

Contour:

$$Y = 1.49 + 2.42X_1 - 0.61X_2 + 0.65X_3 + 0.38X_4 + 1.46X_5 + 0.07X_6 - 0.21X_1^2 - 0.11X_2^2 - 0.27X_3^2 - 0.05X_4^2 - 0.14X_5^2 + 0.04X_6^2 + 0.28X_1X_2 - 0.01X_1X_3 - 0.08X_1X_4 - 0.14X_1X_5 - 0.12X_1X_6 - 0.24X_2X_3 - 0.01X_2X_4 + 0.14X_2X_5 + 0.07X_2X_6 + 0.06X_3X_4 - 0.04X_3X_5 - 0.07X_3X_6 - 0.04X_4X_5 - 0.004X_4X_6 - 0.03X_5X_6$$

Internal:

$$Y = 5.18 + 0.28X_1 + 0.26X_2 + 0.38X_3 + 0.75X_4 + 1.03X_5 + 0.72X_6 - 0.06X_1^2 - 0.10X_2^2 - 0.31X_3^2 - 0.16X_4^2 - 0.19X_5^2 - 0.10X_6^2 + 0.21X_1X_2 + 0.22X_1X_3 - 0.02X_1X_4 - 0.11X_1X_5 - 0.04X_1X_6 - 0.31X_2X_3 - 0.05X_2X_4 + 0.17X_2X_5 - 0.02X_2X_6 + 0.004X_3X_4 + 0.04X_3X_5 - 0.07X_3X_6 + 0.04X_4X_5 + 0.09X_4X_6 - 0.10X_5X_6$$

As in the earlier study (1), the equations were solved for selected levels of the variables to obtain points describing (predicting) performance of batters of that specific composition. Twenty-five related points for all combinations of X_1 and X_2 at the five coded levels gave data for a response surface for those variables. Areas of optimum performance were located by superimposing volume, contour, and internal score responses and establishing limits of acceptable quality for each factor. Since 720 surface permutations exist for each Y_i, it was impractical to compute and construct all the possibilities. In co-operation with the Ohio Agricultural Research and Development Center's Statistical Laboratory, points were generated for 15 selected surfaces, of which seven are presented in Figs. 3 to 9.

These topological drawings indicate the direction and rate of change of each dependent variable with changes in the X_i ratios, considered two-at-a-time while the four remaining ratios were held constant at their normal levels. It must be remembered that the X_i variables are not actual amounts of the stated ingredient, but are the independent ratio of that ingredient to the remainder of the batter as defined above.

In Fig. 3, surface I, A illustrates the effect on cake volume found for simultaneous variation of X_1 (water) and X_2 (sugar) ratios. This surface is a ridge maximizing in the direction of lower sugar concentration. Increasing liquid at any given sugar ratio resulted in increasing volume to a maximum followed by a corresponding reduction at higher liquid levels. B and C show parallel responses of contour and internal scores as a diagonal slope rising in the direction of increased liquid. Neither factor had a maximum within the experimental range. In D, the three individual response surfaces were com-

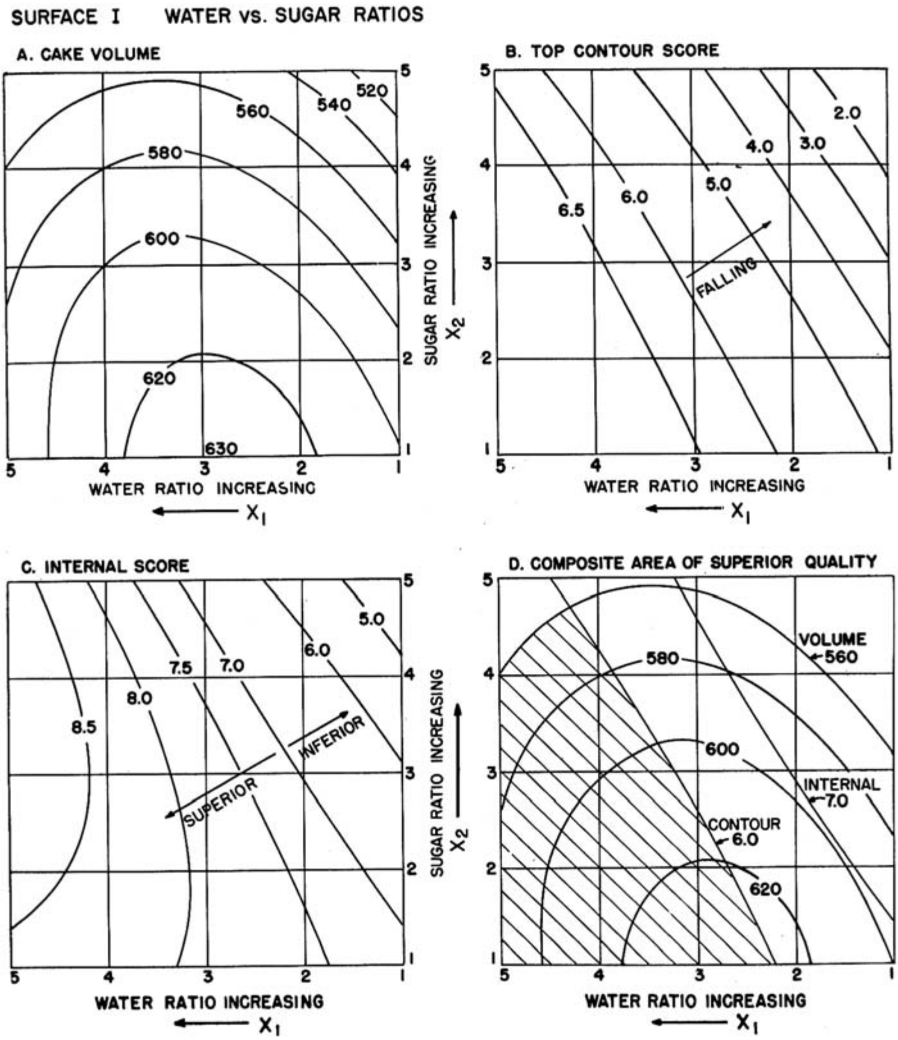
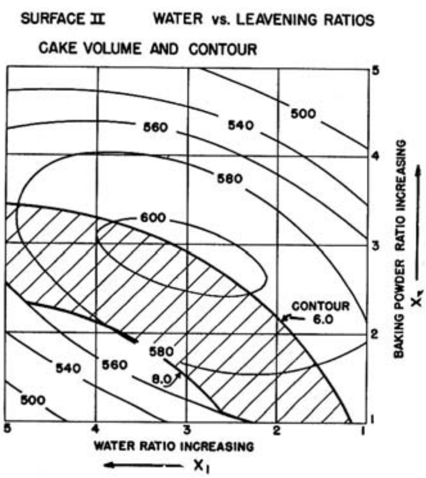


Fig. 3. Contour diagrams representing computed response surfaces generated by simultaneous variation of liquid ratio, X_1, and sugar ratio, X_2, with all other ratios held at the normal (3) level. Surface D indicates the area of over-all acceptability (shaded) defined by the limiting values set upon each Y_i.

bined and the limits of acceptable performance are located by the shaded area which includes only volumes greater than 560 cc., contours between 6.0 and 8.0, and internal scores larger than 7.0.

These figures correlate observations made in previous independent experiments. The critical nature of sugar concentration and its interaction with liquid level are shown. In the area of acceptable quality, higher sugar ratios were compensated only by higher liquid levels and at the expense of reduced cake volume. A paradox of quality optimization is evident from the fact that maximum volume, rounded contour, and maximum internal score do not occur at the same batter composition. A standard observation with liquid level series in the research formula (4) has been that maximum volume coincides with rounded contour (6.5–7.5) scores, but best internal appearance is found in peaked cakes at higher liquid contents. Wilson and Donelson (6) investigated the liquid variable and found that the divergence of quality scores applied both to the research and bakeshop formulations.

The liquid (X_1) and leavening (X_3) ratios are considered together in Fig. 4 (surface II). Cake volume response was a plateau with skewed axes reflecting an interaction of the variables. Volume increased to a maximum and decreased rapidly in the direction of increasing leavening, but was attenuated in the direction of liquid level change. The area of acceptable quality was a narrow band bounded mainly by the limits of contour score with critical tolerance to leavening and greater tolerance to liquid changes at lower leavening ratios.



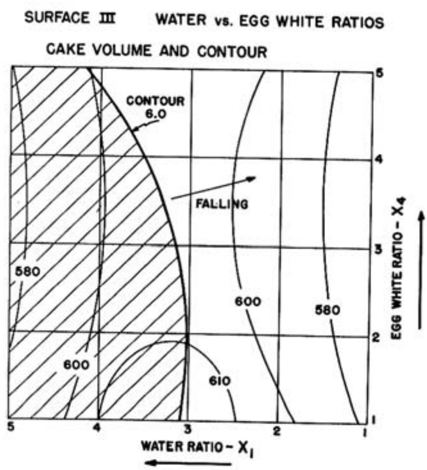
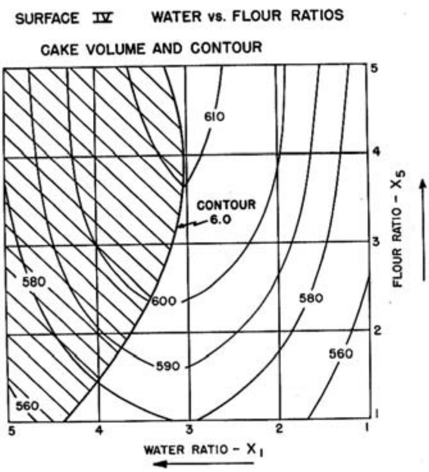


Fig. 4 (left). Response surface generated by simultaneous variation of water ratio, X_1, and leavening ratio, X_3.

Fig. 5 (right). Response surface generated by simultaneous variation of water ratio, X_1, and egg albumen ratio, X_4.

The stabilizing effect of egg albumen is evident in Fig. 5 (surface III) when the X_4 ratio is plotted against the liquid variable. Every point on the volume response surface was greater than the minimum of acceptance, and the rapid curvilinear response to liquid variation noted in Fig. 3 was modified to a gently curving saddle surface. At lower liquid levels sunken contours were predicted at all levels of egg ratio, resulting in the limited area bounded by a contour score of 6.0.

In Fig. 6 (surface IV) the liquid and flour (X_5) variables are considered simultaneously, with volume response appearing as a rising ridge in the direction of increasing flour ratio. The isobars suggest that a maximum may occur beyond the experimental range ($X_5 > 5$) at about the center level of liquid. In this case, both contour and internal score surfaces were slopes rising with increasing liquid ratio, and the area of acceptance was bounded by the lower contour limit.



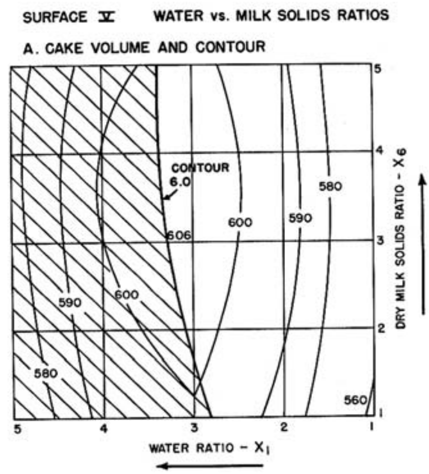


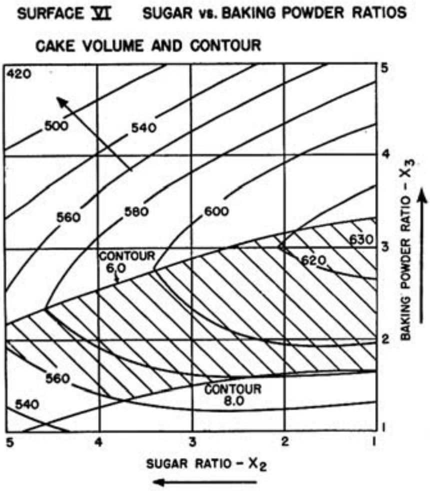
Fig. 6 (left). Response surface generated by simultaneous variation of water ratio, X_1, and flour ratio, X_5.

Fig. 7 (right). Response surface generated by simultaneous variation of water ratio, X_1, and milk solids ratio, X_6.

The liquid and milk solids (X_6) variables in Fig. 7 (surface V) recall the tolerant situation noted for the egg-albumen ratio (Fig. 5). Only slight volume changes were predicted with liquid increases, and essentially no change resulted from milk-solids variation within the experimental range. Again, contour score limited acceptable baking quality to liquid levels greater than the center composition.

When the two critical ratios, sugar and leavening, are considered together in Fig. 8 (surface VI), some of the hazards of formula balancing are made clear. Under the conditions of this experiment, it appeared that sunken contours were predicted for any sugar ratio at most leavening ratios higher than the center composition. However, by reducing the leavening ratio to the coded level-2, tolerance to sugar concentration was increased to contain the full experimental range. This surface suggests that much of the difficulty found in baking at high sugar-flour ratios results from a vanishing tolerance to leavening concentration.

Similarly, in Fig. 9 (surface VII) the stabilizing effect of egg albumen is seen to be ineffectual in preventing sunken cake at sugar ratios above 2.7



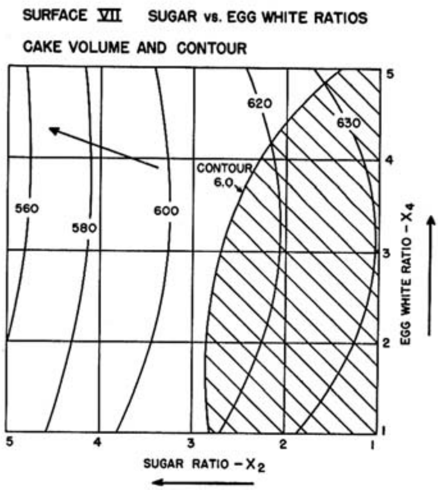


Fig. 8 (left). Response surface generated by simultaneous variation of sugar ratio, X_2, and leavening ratio, X_3.

Fig. 9 (right). Response surface generated by simultaneous variation of sugar ratio, X_2, and egg albumen ratio, X_4.

when leavening and other ratios are at the 3-level. Computations made for a surface in which leavening ratio was held constant at the 2-level indicated contour scores between 7.7 and 6.4 (all rounded) when the sugar ratio increased from 1 to 5 with all other X_i ratios at the 3-level. In this example, a difference in baking powder of only 1.51% (flour basis) accounted for the success or failure of cake quality results.

Practical Application of the Prediction Equations

The ability of the three regression equations to predict baking responses for unknown points *within* the experimental area was tested in an independent experiment. Batters with eight points of interest, plus the design center, were

TABLE V

Comparison of Predicted Cake Performance with Actual Baking Results

Treatment						Cake Volume		Contour Score		Internal Score	
X_1	X_2	X_3	X_4	X_5	X_6	Predicted	Actual	Predicted	Actual	Predicted	Actual
						cc.	*cc.*				
3	1	3	3	3	3	630	610	6.5	7.0	8.0	8.0
4	1	3	3	3	3	618	606	6.7	7.0	8.2	7.5
3	2	3	3	3	3	621	609	6.3	7.0	7.9	7.5
4	4	3	3	3	3	580	561	6.2	6.5	8.2	6.0
5	5	3	3	3	3	537	535	6.5	7.5	8.8	7.5
3.5	3	3	3	5	3	612	610	6.1	7.0	7.3	6.5
3	1	3	3	4	3	633	622	6.4	7.0	7.4	7.5
3	5	3	3	1	3	524	508	2.6	2.0	5.3	3.5

baked in duplicate; a different premium commercial flour was used, and all other ingredients were identical with those employed before. Actual baking data are compared with computed values in Table V. In this series, good agreement with predicted values was obtained for volume, contour, and most internal scores, although a different flour was used. Eleven batters with one or more ratios outside the experimental range were also tested. In every case, computed layer volumes were lower than actual baking results (mean = – 54 cc.) and both contour and internal scores averaged below the actual values. Thus, precision of the method was reduced when extrapolated beyond the area of study, but errors were in a conservative direction.

Mathematical Transformations

The illustrated surfaces represent only 1.0% of the possible two-at-a-time arrangements of the variables. The problems of analyzing and interpreting the mass of possible information are still largely unsolved. Exploration of any surface may suggest other directions to search for optimum cake quality. The question is: where to begin, and how to reject unprofitable surfaces *a priori.*

Box (3), Chang *et al.* (10), and Davies (11) suggest location of stationary points for each multiple regression equation and subsequent transformation to canonical coefficients. A stationary point is uniquely defined when all slopes equal zero and the point is, therefore, a minimum, maximum, or min.-max. for the system. Coordinates for such points are found by simultaneous solution of a series (six, in this case) of equations obtained from the original regression equation by partial differentiation with respect to each X_i in turn.

The partial equations for cake volume (in two decimals) are:

$$\partial Y/\partial X_1 = 7.64 - 17.32X_1 + 2.84X_2 + 13.26X_3 + 0.16X_4 - 0.46X_5 - 0.15X_6.$$
$$\partial Y/\partial X_2 = 15.94 + 2.84X_1 - 6.70X_2 - 8.40X_3 - 0.99X_4 + 1.66X_5 + 0.04X_6.$$
$$\partial Y/\partial X_3 = 106.82 + 13.26X_1 - 8.40X_2 - 39.57X_3 - 1.26X_4 - 0.66X_5 - 2.79X_6.$$
$$\partial Y/\partial X_4 = - 0.90 + 0.16X_1 - 0.99X_2 - 1.26X_3 + 3.43X_4 - 0.14X_5 - 0.57X_6.$$
$$\partial Y/\partial X_5 = 24.72 - 0.46X_1 + 1.66X_2 - 0.66X_3 - 1.04X_4 - 3.94X_5 - 1.07X_6.$$
$$\partial Y/\partial X_6 = 20.92 - 0.15X_1 + 0.04X_2 - 2.79X_3 - 0.57X_4 - 1.07X_5 - 1.94X_6.$$

Two similar sets of equations were derived from the contour and internal score equations. Simultaneous solution of the sets by computer resulted in the coordinates for the stationary points summarized in Table VI.

TABLE VI
COORDINATES OF X_i VARIABLES AT STATIONARY POINTS
(Coded levels)

Response Y	Coded Ingredient Ratio					
	X_1 Water/R	X_2 Sugar/R	X_3 Leavening/R	X_4 Egg White/R	X_5 Flour/R	X_6 DMS/Short.
Volume	2.8744	– 0.0273	3.3121	2.9553	3.8235	2.8142
Contour	– 0.8602	– 4.8866	3.2240	6.3845	1.3695	5.5243
Internal score	– 3.6971	1.3652	– 1.7901	4.3403	3.3742	5.3320

Inspection of these coded levels, in view of the responses discussed previously, suggested superior (maximum) performance at the co-ordinates for volume. Interpretation of contour and score co-ordinates were less certain, since several negative ratios and off-scale positive points were indicated. Performance at the internal-score stationary point was especially suspect, since the leavening ratio is highly negative and required a negative weight of baking powder to fulfill the treatment. Minimum cake performance was expected in this case.

Actual baking responses were compared with those predicted from the above co-ordinates by decoding ratios and solving for ingredient quantities. Results from these baking comparisons are listed in Table VII. As expected superior volumes and satisfactory contour and crumb were obtained at the volume stationary point, although crust color was very pale with this combination of ingredients. An exceptionally large, very rounded cake resulted at

TABLE VII
COMPUTED VS. ACTUAL RESPONSES OF THE Y-VARIABLES AT EACH STATIONARY POINT

Stationary Point	Y-Response at Stationary Point					
	Volume		Contour		Score	
	Predicted	Actual	Predicted	Actual	Predicted	Actual
	cc.	cc.	units	units	units	units
For volume	635	628	6.0	7.0	6.9	8.0
For contour	533	647	5.5	7.5	7.7	9.5
For internal score	245	375	−2.9	0.0	15.7	11.5

the contour stationary point, although indifferent or unsatisfactory volume was predicted by extrapolation to the off-scale points. Both expected and actual cake were failures at the internal score point, owing to the indicated omission of leavening. The logic of this result is seen in the maximization of internal score at the score stationary point. In the present scoring system a larger number indicates finer cell structure. Maximum fineness of cells occurs in dense, unleavened batters, and the stationary point located this area although the information was of little value.

Preliminary efforts to relocate the co-ordinate axes at the stationary points by canonical transformation have failed to give useful results. Such mathematical treatment is considered of marginal value for those responses which require extrapolation from the experimental range. On the other hand, the stationary co-ordinates for cake volume (Table VI) may be an interesting design center for the study of batter systems with large volume potential and low sugar level.

The few preceding examples of response surfaces have shown that no single optimum combination of ingredients exists in the cake-batter complex. There are, rather, areas of satisfactory performance within the experimental range, varying in size and shape according to the limits of acceptability imposed by the investigator. This paper can only suggest the large amount of information resulting from a relatively small, controlled multiple-factor experimental design, as applied to baking response and formulation problems.

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