MULTI-FACTOR RESPONSES OF CAKE QUALITY TO BASIC INGREDIENT RATIOS[1]

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ABSTRACT

The main and interaction effects of ingredient variation on baking quality are difficult to determine by single-factor experimental procedure. An analysis of these effects on volume and contour of lean-formula white layer cakes was obtained, using a central composite design of the Box-Wilson multiple-response surface type. Amounts of five basic formula ingredients were transformed to a series of four linearly independent ratios, variation increments were established, and batters representing combinations of the ratios were baked to fulfill the requirements of the design.

Statistical analysis of layer-volume and top-contour scores provided multiple regression equations describing the quality responses to simultaneous variation of the ratios. A series of response surface drawings illustrate the significant ingredient-ratio effects and the areas of formulation in which both volume and contour are acceptable.

The effects of baking powder and sugar/water ratios were curvilinear and critical, with optimum volume and contour occurring at a narrow range of the variables passing through the center of the design. A strong interaction resulted in markedly reduced quality if leavening, sugar, and water were out of balance. The main effect of the shortening ratio was linear and nonsignificant, but significant interactions were found with flour and sugar/water ratios. Quadratic and all interaction effects of the flour ratio were significant and resulted in larger than normal cake volume at higher levels of the variable.

A suitable test baking formula is usually developed by varying one ingredient at a time while other components are held "constant." This approach, along with empirical rules for balancing formulas (8), has been the basis for many of the reported ingredient-cake quality studies. These balancing methods, if skillfully applied, are effective in solving bakeshop and production problems, but result in serious limitations when fundamental ingredient effects are to be studied.

Baked products result from ingredient variables which cannot be assumed to act independently as implied by single-factor experiments. The effect on a baked product of variations in ingredient A in a batter composed of $A + B + C + D + n = 100.00\%$ may depend on the specific levels chosen for B, C, D, and n. In piecemeal experiments ingredient-interaction effects are difficult to isolate and results of variation series may not be directly comparable.

This paper presents results from a central composite design of the

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Box-Wilson multiple response surface type (2,3) applied to the problem of basic ingredient ratios vs. cake quality. Donelson and Wilson (5) employed this method to study the effect of relative quantity of flour fractions on cake quality by means of three independent variables. These authors include an introduction to multi-factor designs and a bibliography of basic references (1954–1958).

Baird and Mason (1) developed multi-variable equations to describe fertility vs. corn yield response surfaces and illustrate the use of contour diagrams. Dechman and Van Winkle (4) found agreement within the experimental range between a 4^4 complete factorial and a 4 factor (2^4) Box design which used about one-tenth the number of the experimental observations. Kern and Kenworthy (6) presented a strong case for multi-factor designs applied to formula optimization in the chemical engineering field.

Materials and Methods

The cake method and ingredients used in this study were essentially as described by Kissell (7) in the lean white layer formula. A commercially milled and bleached short-patent flour, previously tested at normal lean-formula proportions, was used throughout. Commercial double-action baking powder and high-ratio emulsified shortening were used, along with a 69% sugar solution prepared from fine granulated sucrose. Mixing schedule and depositing procedure were as described, except that baking time was increased to 22 minutes at 375°F.

To study systematically the relative contribution of each basic ingredient to cake quality, the lean-formula composition (Table I)

TABLE I
NORMAL COMPOSITION OF LEAN CAKE FORMULA

Ingredient	Flour Weight Basis	Total Batter Basis	Quantity
	%	%	g
A Baking powder	4.7	1.29	7.1
B Shortening	27.9	7.62	41.8
C Flour	100.0	27.35	150.0
D Sugar	130.0	35.56	195.0
E Water	103.0[a]	28.18	154.5
Total		100.00	548.4

[a] For the specific flour used; optimum liquid level varies with different flours.

was used as the center point of the composite design. Since this method was developed originally by single-factor variation procedure, a secondary result of the experiment would indicate how well optimum formula balance had been achieved.

Experimental Design. This study, based on the hypothesis that cake volume, appearance, and other quality factors are functionally related to the specific batter composition, attempts to fit multiple-regression equations describing the quality-composition responses. In addition to separate studies of response surfaces for cake volume-batter composition and top contour-composition, a graphical method is used to indicate the areas of composition at which acceptable volume and appearance occur simultaneously.

The composition of a lean-formula cake batter has the form: A (baking powder) + B (shortening) + C (flour) + D (sugar) + E (water) = 100.00%. This equation implies mathematical linear dependence of variables if the amounts of ingredients are used directly as variables, since from the equation the quantity of any ingredient is uniquely determined by the amounts of the other four. To function in a multiple-factor analysis, these ingredients may be transformed to ratios which can be varied independently. Care must be taken to ensure the independent operation of the ratios, since an error in the choice of variables may lead to indeterminate solutions. For this experiment the following ingredient ratios were selected as the x_i variables:

$$\begin{aligned}
x_1 &= \text{baking powder}/(\text{shortening} + \text{flour} + \text{sugar} + \text{water})\\
x_2 &= \text{shortening}/(\text{flour} + \text{sugar} + \text{water})\\
x_3 &= \text{flour}/(\text{sugar} + \text{water})\\
x_4 &= \text{sugar}/\text{water}
\end{aligned}$$

Selecting values for each of the independent ratios uniquely determines the amounts of the five ingredients. The constraint of transforming to ratios complicates interpretation of results but permits a mathematical approximation to the functional cake quality-batter composition relationship. The equations were obtained by fitting a second-order polynomial in four variables by Taylor series expansion, with all higher-order terms considered as a negligible remainder, Rx_i:

$$\begin{aligned}
y = {} & \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{44} x_4^2 + \\
& \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{14} x_1 x_4 + \beta_{23} x_2 x_3 + \beta_{24} x_2 x_4 + \beta_{34} x_3 x_4 + Rx_i
\end{aligned}$$

In statistical analysis the cross product terms measure ingredient interactions, and the remainder provides a lack-of-fit term.

A center point for the design should be selected with ingredients at levels expected to yield at least satisfactory experimental results. In this example the lean cake formulation provided a convenient departure point. With the center composition selected, the normal x_i ratios were calculated by using the normal weight composition of the lean formula (Table I). The design depends upon a symmetrical selection of variation increments about the center composition. These levels of variation were chosen to be within the range of reasonable formulations, and the increments were carefully selected, since interpretation of the results is valid only within the experimental limits. The extrapolation of a polynomial surface is not trustworthy, and indicated optima outside the experimental region must be checked in the laboratory.

For convenience of notation and solving for coefficients in the matrix, the actual x_i ratios were coded in whole integers. Table II lists the experimental increments, actual ingredient ratios corresponding to the coded levels, and equations for coding and decoding the ratios.

TABLE II

EXPERIMENTAL INCREMENTS, VALUES OF CODED LEVELS, AND EQUATIONS RELATING ACTUAL x_i AND CODED X_i RATIOS

x_i Variable	± Increment	X_i Coded Level				
		1	2	3 (Normal)	4	5
x_1	0.003	0.007	0.010	0.013	0.016	0.019
x_2	0.009	0.066	0.075	0.084	0.093	0.102
x_3	0.030	0.369	0.399	0.429	0.459	0.489
x_4	0.185	0.892	1.077	1.262	1.447	1.632

Where:

$$X_1 = \frac{x_1 - 0.004}{0.003} \qquad X_3 = \frac{x_3 - 0.339}{0.030}$$

$$X_2 = \frac{x_2 - 0.057}{0.009} \qquad X_4 = \frac{x_4 - 0.707}{0.185}$$

In this system, level 3 represents the normal ratios of ingredients in a lean-formula batter, 1 and 2 levels are the indicated amounts below the normal ratios, and 4 and 5 are higher than normal.

The experiment involved a 2^4 factorial for all combinations of the 2 and 4 levels, plus each X_i variable in turn at the 1 and 5 level while the remaining variables were held at the 3 (normal) level. In all, 20 replications of the center treatment, four replications of each 2- and 4-level combination, and two replications of each 1- and 5-level treatment were randomized throughout 10 days of baking. This scheme is a modification of the general plan outlined by Cochran and Cox (2).

Practical Considerations. Before this type of experiment can be performed, the coded X_i ratios for each treatment must be translated into working quantities of ingredients. Batter compositions were obtained by systematic algebraic solutions for A, B, C, D, and E in terms of actual x_i ratios and a unit quantity of product. Equations derived for the general case follow:

$$A=\frac{x_1}{1+x_1};\qquad B=\frac{x_2\,(1-A)}{1+x_2};\qquad C=\frac{x_3\,(1-A-B)}{1+x_3};$$

$$D=\frac{x_4\,(1-A-B-C)}{1+x_4};\qquad E=\frac{D}{x_4}.$$

The resulting percentages for each ingredient are summarized by treatment in Table III, along with average layer volume, top-contour score, and a key for descriptive decoding of the scores. Subjective

TABLE III

TREATMENT COMBINATIONS WITH CORRESPONDING BATTER COMPOSITIONS, CAKE VOLUMES, AND TOP-CONTOUR SCORES

Treatment Coded Level				Batter Composition					Cake Volume	Top-Contour Score[b]
				A	B	C	D	E		
X_1	X_2	X_3	X_4	Baking Powder	Shortening	Flour[a]	Sugar	Water		
				%	%	%	%	%	cc	
3	3	3	3	1.28	7.65	27.34	35.56	28.17	571	6.9
2	2	2	2	0.99	6.91	26.26	34.14	31.70	489	7.8
4	2	2	2	1.57	6.87	26.11	33.94	31.51	556	7.3
2	4	2	2	0.99	8.43	25.83	33.58	31.17	483	7.8
4	4	2	2	1.57	8.37	25.68	33.38	31.00	562	8.0
2	2	4	2	0.99	6.91	28.98	32.73	30.39	502	7.3
4	2	4	2	1.57	6.87	28.81	32.54	30.21	586	7.3
2	4	4	2	0.99	8.43	28.50	32.19	29.89	505	7.3
4	4	4	2	1.57	8.37	28.33	32.01	29.72	583	8.8
2	2	2	4	0.99	6.91	26.27	38.92	26.91	529	4.0
4	2	2	4	1.57	6.87	26.11	38.70	26.75	463	1.5
2	4	2	4	0.99	8.43	22.94	40.00	27.64	472	3.8
4	4	2	4	1.57	8.37	22.80	39.78	27.48	416	1.0
2	2	4	4	0.99	6.91	28.97	37.33	25.80	565	5.8
4	2	4	4	1.57	6.87	28.81	37.11	25.64	514	2.0
2	4	4	4	0.99	8.43	28.50	36.71	25.37	557	5.8
4	4	4	4	1.57	8.37	28.33	36.50	25.23	512	2.0
1	3	3	3	0.70	7.69	27.50	35.77	28.34	493	7.5
5	3	3	3	1.86	7.60	27.18	35.35	28.01	528	3.0
3	1	3	3	1.28	6.12	27.80	36.15	28.65	565	6.5
3	5	3	3	1.28	9.14	26.89	34.97	27.72	570	7.5
3	3	1	3	1.28	7.65	24.54	37.12	29.41	532	6.5
3	3	5	3	1.28	7.65	29.91	34.12	27.04	595	7.0
3	3	3	1	1.28	7.65	27.34	30.05	33.68	490	7.5
3	3	3	5	1.28	7.65	27.34	39.52	24.21	454	1.5

[a] 14% moisture basis.

[b] Key to top-contour appearance: 10 = highly peaked; 9 = peaked; 8 = rounded-peaked; 7 = rounded-normal; 6 = rounded-flat; 5 = very slightly fallen; 4 = sligntly fallen; 3 = fallen; 2 = greatly fallen; 1 = extremely fallen.

judgments of internal appearance (cell size, cell wall thickness, and crumb uniformity) and penetrometer measurements of crumb structure were also analyzed; the results are beyond the scope of this paper.

One complication in the present study resulted from use of the lean-formula method which introduces sugar (D) as a 69% solution. It was necessary to calculate the volume of solution containing the required dry-weight of sugar and the amount of water in that volume for each treatment. Lean-formula procedure was used regardless of batter consistency. A constant batter weight assured sufficient material of all treatments for two 240-g. aliquots per replication and the equivalence of each aliquot to the total batter composition.

Results and Discussion

Cake Volume vs. Batter Composition. Table IV lists each partial regression coefficient (b_i) along with its standard deviation and t-test for significance. Of the linear effects only X_1 (baking powder ratio) and X_4 (sugar/water ratio) were highly significant. All quadratic terms were significant to at least the 5% level, X_1^2 and X_4^2 having the largest effects. Considering the interaction terms, X_1X_4 was most significant, although all interactions with X_4 and X_2X_3 (shortening ratio by flour ratio) were significant at the 1% level. When the b_i coefficients are substituted in the general multiple regression equation (Table IV), the expected cake volume for any combination of coded

TABLE IV

PARTIAL REGRESSION COEFFICIENTS (b_i), STANDARD ERRORS (s_b), AND t-TEST OF SIGNIFICANCE FOR CAKE VOLUME

VARIABLE	b_i	s_b	t
Linear: X_1	195.244	10.62	18.38**
X_2	11.581	10.62	1.09
X_3	−15.206	10.62	1.43
X_4	232.119	10.62	21.85**
Quadratic: X_1^2	−17.100	1.33	12.86**
X_2^2	− 2.850	1.33	2.14*
X_3^2	− 3.850	1.33	2.90**
X_4^2	−26.663	1.33	20.05**
Interaction: X_1X_2	1.406	1.33	1.06
X_1X_3	2.688	1.33	2.02*
X_1X_4	−32.875	1.33	24.72**
X_2X_3	5.719	1.33	4.30**
X_2X_4	− 7.094	1.33	5.33**
X_3X_4	11.313	1.33	8.51**
b_0	−76.810		

Multiple regression equation for coded values of X_i:

$$Y \text{ (Cake volume)} = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_{11}X_1^2 + b_{22}X_2^2 + b_{33}X_3^2 + b_{44}X_4^2 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{34}X_3X_4.$$

Standard error of estimate		=	± 10.64 cc.
Confidence interval at 5% level (d.f = 85)		=	± 21.16 cc.
Correlation index	R^2	=	0.9551
Multiple correlation coefficient	R	=	0.9773**

X_i ratios can be calculated. The standard error of any estimated point on a response surface is ± 10.64 cc. or, in terms of 5% confidence interval with 85 d.f., ± 21.16 cc. The multiple correlation coefficient, R, is a measure of fit of experimental data to the calculated surface and was highly significant at 0.9773. The equation accounted for 95.5% of the variation in cake volume, since the correlation index, R^2, equals 0.9551.

By solving the multiple regression equation for selected coded values of X_i, estimated cake volumes were calculated for the 625 possible combinations of the variables. These estimated volume points provided the basis for response-surface models, isometric drawings, or surface-contour diagrams to facilitate explanation of the effect of batter variation on cake quality.

Layer Appearance (Top-Contour Score) vs. Batter Composition. Similar calculations for top-contour scores are summarized in Table V. Again, the variable, its partial regression coefficient, standard error, and t-test statistic are included. The values of b_i may again be substituted in the general coded equation (Table V) for solution of estimated response surface points representing layer appearance.

From the analysis it was again evident that those treatments

TABLE V

PARTIAL REGRESSION COEFFICIENTS (b_i), STANDARD ERRORS (s_b), AND t-TEST OF SIGNIFICANCE FOR TOP-CONTOUR SCORE

VARIABLE	b_i	s_b	t
Linear: X_1	4.544	0.642	7.08**
X_2	0.406	0.642	0.63
X_3	−0.194	0.642	0.30
X_4	4.206	0.642	6.55**
Quadratic: X_1^2	−0.500	0.080	6.22**
X_2^2	−0.063	0.080	0.78
X_3^2	−0.125	0.080	1.56
X_4^2	−0.688	0.080	8.56**
Interaction: X_1X_2	0.125	0.080	1.56
X_1X_3	−0.031	0.080	0.39
X_1X_4	−0.875	0.080	10.89**
X_2X_3	0.094	0.080	1.17
X_2X_4	−0.188	0.080	2.33*
X_3X_4	0.344	0.080	4.28**
b_0	−2.823		

Multiple regression equation for coded values of X_i:

$$Y\text{ (Contour score)} = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_{11}X_1^2 + b_{22}X_2^2 + b_{33}X_3^2 + b_{44}X_4^2 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{34}X_3X_4.$$

Standard error of estimate		= ± 0.64
Confidence interval at 5% level (d.f. = 85)		= ± 1.28
Correlation index	R^2	= 0.9357
Multiple correlation coefficient	R	= 0.9673**

involving X_1 or X_4 had the greatest effect on top contour. The X_1X_4 interaction was particularly large, and both X_2 and X_3 ratios had significant interactions with X_4. This indicates that variations in some of the factors causing significant changes in cake volume produce concomitant changes in top contour. The standard error of estimate was ±0.64 score units, which at the 5% confidence level equals ±1.28 units. The fit of experimental data to the regression surface was again very good, with the equation accounting for 93.6% of the variation in top-contour score ($R^2 = 0.9357$).

Cross-sections of cakes representing nine of the treatment combinations are shown in Fig. 1. These illustrate the effect of varying each X_i ratio from 1, the lowest coded level, to 5, the highest, while holding all other ingredient ratios at the normal or 3 level. The main effect of each ingredient is suggested by the horizontal rows of cakes. Layers in the center vertical row were produced when all X_i ratios equaled 3 (i.e., at the center of the design).

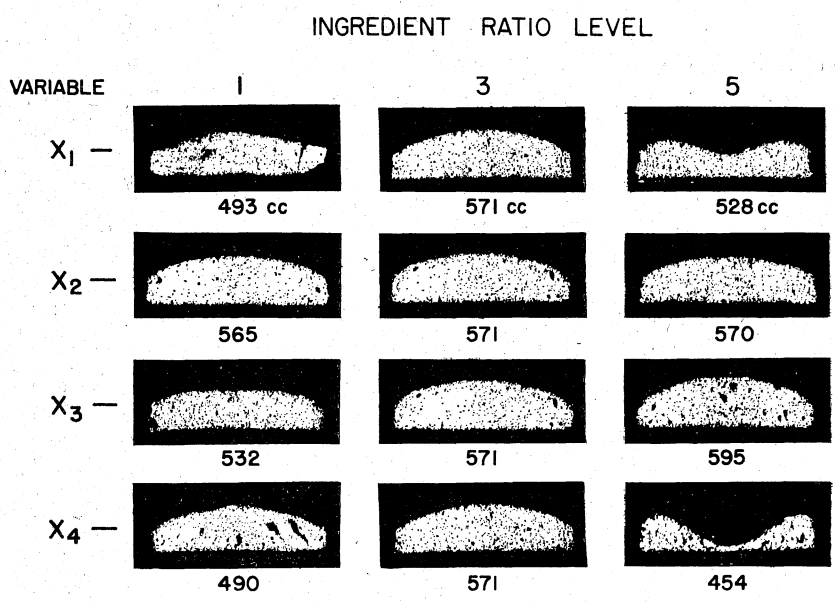

Fig. 1. Layer cross-sections showing the main effects of each indicated X_i variable at low (1), normal (3), and high (5) coded levels, with remaining variables fixed at 3. X_1 = baking powder ratio; X_2 = shortening ratio; X_3 = flour ratio, and X_4 = sugar/water ratio.

If the baking-powder ratio, X_1, is increased from 1 to 5 while the remaining variables are held at the 3 level, cake volume and appearance are affected greatly. Layer volume, whether due to peaking and shrinkage or to falling, was reduced on either side of an optimum ratio, which seems to be the 3 level. Similar variations in the shortening ratio, X_2, indicated no significant change in volume or appearance. Increasing the flour ratio, X_3, resulted in significantly increasing cake volume and a tendency toward peaking within the experimental range of the variable. Finally, the effect of increasing the sugar/water ratio, X_4, was analogous to the response shown by X_1, with inferior volumes and appearance resulting on either side of an optimum level.

These visual effects of ingredient variation on cake quality represent the linear and quadratic terms referred to in Tables IV and V. These effects, and the equally important quality responses to ingredient interaction, are presented in a series of graphs. Since the representation of four simultaneous variables in a five-dimensional space is obviously impossible, the variables have been considered as increasing two-at-a-time, with the remaining ratios held fixed at the normal level. A large number of other surfaces exist and could be illustrated by holding the remaining variables at other levels (i.e. both fixed at 1, 2, 4, or 5; each fixed at combinations of 1 through 5).

Two alternative forms of response-surface representation are compared in Fig. 2. Drawings A and B are isometric projections suggesting a three-dimensional surface, reflecting the respective response of cake volume and top contour to simultaneous changes in baking powder, X_1, and shortening, X_2, ratios with the remaining variables held at the 3 level. This type of drawing, although informative in a qualitative sense, has several limitations. It is difficult to locate points on the surface with respect to base coordinates, and areas of equal performance are not defined precisely.

Contour diagrams analogous to topographical contour maps have been proposed as showing better relationships of surface to base. Corresponding contour diagrams for the respective cake volume and top-contour surfaces are given as C and D. This form defines the surface, indicates areas of surface maxima, suggests changes of slope by relative spacing of contour lines, and locates points on the surface with respect to coded levels of the X_i variables.

If limits of acceptability are assigned to the dependent variables and the contour drawings, C and D, superimposed, the area of coincidence of two or more quality factors is defined. In drawing Fig. 3, A, it was assumed that cake volume of 540 cc. or larger and top-contour scores between 6 (rounded-flat) and 8 (rounded-peaked) are acceptable. Under these restrictions, the cross-hatched area contains the only values of leavening and shortening ratios that are expected to yield cake of acceptable quality. In this and subsequent drawings, the main

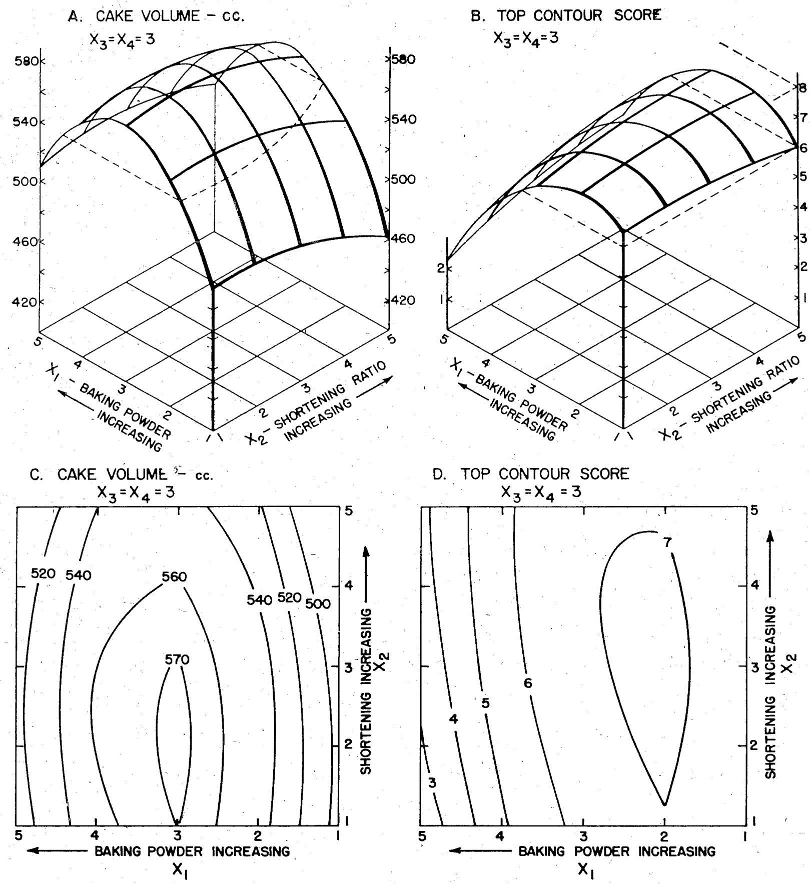

Fig. 2. A and B, isometric drawings of response surfaces attributed to simultaneous variation of baking-powder ratio, X_1, and shortening level, X_2. C and D, corresponding contour diagrams with isobars connecting points of equal cake volume or appearance.

isobaric lines shown are those for calculated layer volume. Broken lines indicate the limiting value of top-contour score, with an arrow in the direction of decreasing quality.

Both cake volume and top contour are markedly affected by changes in baking-powder level, regardless of the level of shortening ratio. Conversely, increasing the shortening ratio within the experimental range results in very little volume or contour response, although there is a tendency (statistically nonsignificant) toward interaction at higher levels of X_2.

Superimposed contour diagrams show the effect of simultaneous

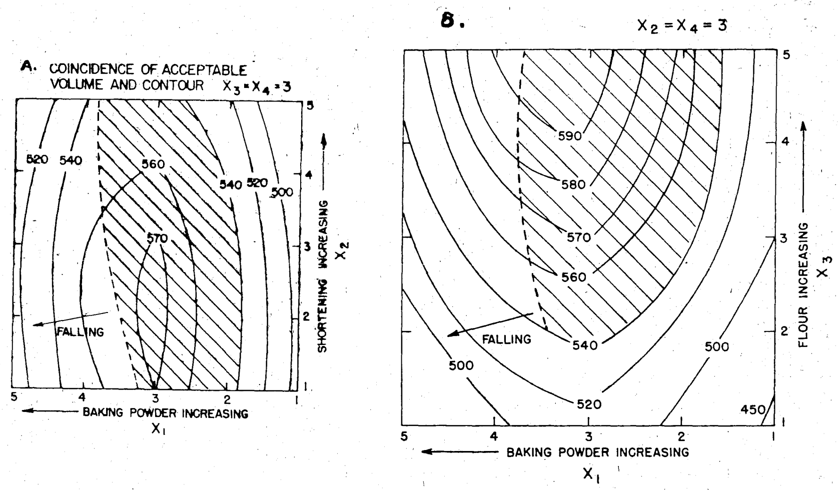

Fig. 3. A, composite contour drawing with cross-hatched area of over-all acceptable quality limited by volume = 540 cc. or greater and top-contour score between 6 and 8. B, similar contour diagram of cake quality responses to variation of baking powder, X_1, and flour, X_3, ratios.

variation of baking powder, X_1, and flour ratio, X_3, in Fig. 3, B. The surface attributed to cake volume is a rising ridge generated by a rapid volume increase and subsequent decrease as X_1 increases, and by a gradual volume increase as X_3 increases. The isobaric lines for

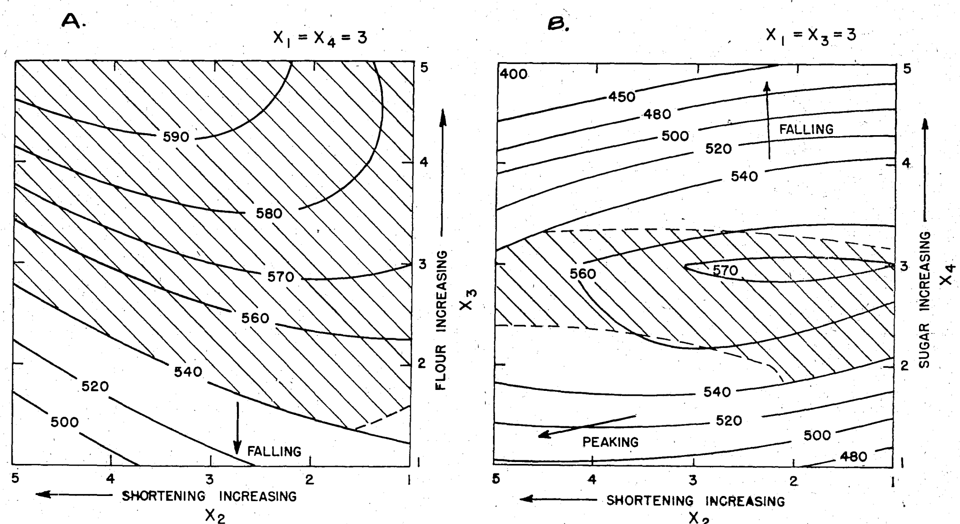

Fig. 4. A, composite contour diagram of cake quality responses to variation of baking powder, X_2, and flour, X_3, ratios. B, diagram of cake quality responses to variation of baking powder, X_2, and sugar/water, X_4, ratios.

cake volume suggest that a maximum point was not included in the experiment, but may exist at coded levels of X_1 greater than 3 and X_3 greater than 5. This case illustrates the utility of response surface exploration in suggesting the direction of further experimentation if the attainment of maximum yield (or optimum conditions) is desired. The limiting value of top-contour score defines an area which appears to be critical with respect to changes in X_1 and relatively tolerant to the level of X_3. Interaction effects of these variables are not highly significant.

If the variables with least significant main effects, X_2 and X_3, are considered together, as in Fig. 4, A, a tolerant situation results. The volume surface is again a form of rising ridge with a maximum beyond the experimental range ($X_3 > 5$). Both quadratic and interaction terms are significant for volume, but none are significant for top-contour score. Thus, acceptable appearance would be expected throughout most of the range of the variables, and the gently rising slope due to increasing X_3 results in a large area of over-all acceptability when X_1 and X_4 (sugar/water ratio) are held at their normal levels.

A very different situation is illustrated in Fig. 4, B, where X_2 and X_4 are considered simultaneously. Here a unit change in the X_2 ratio gives a nonsignificant volume response, but a unit change in X_4 produces large effects in both volume and contour. The area of coincident

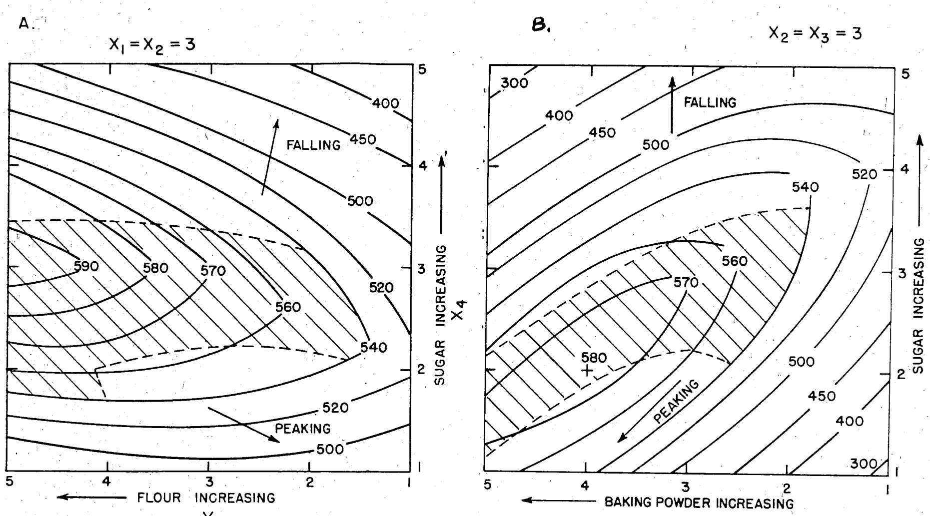

Fig. 5. A, composite contour diagram of cake quality responses to variation of flour, X_3, and sugar/water, X_4, ratios. B, diagram of cake quality responses to variation of baking powder, X_1, and sugar/water, X_4, ratios.

acceptability is again a narrow band indicating tolerance to shortening level variation but highly critical to the sugar/water concentration. Significant interaction terms for these variables indicate that the volume responses and, to a lesser extent, contour responses due to changes in X_4 depend upon the specific level of X_2. In the case of X_4, the specific amount of water is confounded with the amount of sugar so that as sugar increases, water must decrease. This unavoidable restriction on the variable had the effect of magnifying the critical response to X_4. By defining a different set of x_i variables the effects of sugar and water levels could be separated, although *some* two ingredients would always be confounded in the new x_4 ratio.

A relative tolerance of cake volume to flour ratio, X_3, and the critical response to X_4 are evident in Fig. 5, A. Highly significant interaction terms were found for both volume and contour, indicating that the response to one variable is conditioned by the level of the other. In all cases involving combinations with sugar/water ratio, reduced layer volume is accompanied by either extreme falling or peaking, as indicated by the arrows.

The final two-at-a-time combination of variables is plotted in Fig. 5, B. As predicted by the highly significant t-values for linear, quadratic, and interaction terms (Tables IV and V), the relationship of X_1 and X_4 defines a very critical surface. It may be described as a diagonal standing ridge with a maximum near $X_1 = 4$ and $X_4 = 2$. Since the indicated volume maximum occurs near the expected threshold of peaking (broken line), a cake at this point may not be of optimum quality. These variables have a narrow range of tolerance and the response of one is highly dependent on the level of the other. Photographs of cakes corresponding to five points on this surface are shown in Fig. 1. They include the following coordinate points:

$X_1 = 1$	$X_4 = 3$	(volume = 493 cc., peaked)
$X_1 = 5$	$X_4 = 3$	(volume = 528 cc., fallen)
$X_1 = 3$	$X_4 = 1$	(volume = 490 cc., peaked)
$X_1 = 3$	$X_4 = 5$	(volume = 454 cc., greatly fallen)
$X_1 = X_4 = 3$ (center point)		(volume = 571 cc., rounded)

Testing Estimated Cake Performance. The following sequence may be used to check the baking performance of any selected combination of X_i variables. Select a point of interest on a surface (e.g., Fig. 5, A: $X_1 = X_2 = 3$, $X_3 = 5$, and $X_4 = 1$) and decode the ratios by solution of the equations relating X_i and x_i (Table II):

$$x_1 = 0.003\,(X_1) + 0.004 = 0.003\,(3) + 0.004 = \mathit{0.013}$$
$$x_2 = 0.009\,(X_2) + 0.057 = 0.009\,(3) + 0.057 = \mathit{0.084}$$
$$x_3 = 0.030\,(X_3) + 0.339 = 0.030\,(5) + 0.339 = \mathit{0.489}$$
$$x_4 = 0.185\,(X_4) + 0.707 = 0.185\,(1) + 0.707 = \mathit{0.892}$$

Amounts of ingredients satisfying these x_i ratios are found by systematic solution of the equations for A, B, etc.

$$A \text{ (baking powder)} = \frac{x_1}{1+x_1} = \frac{0.013}{1.013}(100) = 1.28\%$$

$$B \text{ (shortening)} = \frac{x_2\,(1-A)}{1+x_2} = \frac{0.84\,(1-0.0128)}{1+0.084}(100) = 7.65\%$$

Similarly:

C (flour) = 29.91% D (sugar) = 28.83% E (water) = 32.33%

A + B + C + D + E = 100.00 = total batter.

This batter composition was tested with the following results: actual cake volume = 510 cc.; predicted = 475 cc.; deviation = +35 cc. Actual top score = 7.5; predicted score = 7.0; deviation = +0.5 unit.

Although the volume deviation was somewhat larger than the 21.16 cc. confidence interval, top contour was close to the expected and overall baking results confirm the prediction for the lower left corner of Fig. 5, A.

In this report some response surface drawings were given as visual aids, although much more information was obtained from the Box-Wilson analysis. In addition, 144 surfaces exist if all combinations of the variables are held two-at-a-time at all possible 1 to 5 levels. A series of these surfaces, holding the same variables at different levels, are generically related through the regression equations and indicate the shifts of quality factors with respect to formulation changes.

In more complex experiments (e.g. adding milk and egg albumen to the formula) some mathematical techniques would aid the interpretation of results. Partial differentiation of the regression equation with respect to each variable in turn was studied by Donelson and Wilson (5). Davies (3) and Cochran and Cox (2) suggest simplification of the fitted quadratic surface by transforming the equation to its canonical form.

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Literature Cited

1. Baird, B. L., and Mason, D. D. Multi-variable equations describing fertility-corn yield response surfaces and their agronomic and economic interpretation. Agron. J. **51:** 152–156 (1959).
2. Cochran, W. G., and Cox, Gertrude M. Experimental designs (2nd ed.), chap. 8A. Wiley: New York (1957).

3. DAVIES, O. L. The design and analysis of industrial experiments (2nd ed. rev.), chap. XI. Hafner Pub. Co.: New York (1956).
4. DECHMAN, D. A., and VAN WINKLE, M. Perforated plate column studies by the Box method of experimentation. Ind. Eng. Chem. **51:** 1015–1018 (1959).
5. DONELSON, D. H., and WILSON, J. T. Effect of the relative quantity of flour fractions on cake quality. Cereal Chem. **37:** 241–262 (1960).
6. KERN, D. Q., and KENWORTHY, O. O. A farewell to the cookbook. Ind. Eng. Chem. **52:** 42A–47A (1960).
7. KISSELL, L. T. A lean-formula cake method for varietal evaluation and research. Cereal Chem. **36:** 168–175 (1959).
8. PYLER, E. J. Baking science and technology, vol. II, chap. 24. Siebel Pub. Co.: Chicago (1952).